CH 12 MOVING HORIZON ESTIMATION (MHE)

AIM: GIVEN MEASUREMENTS $y_0, \dots, y_{N-1}$
ESTIMATE STATES $x_0, \dots, x_N$

$$\boxed{\begin{aligned} x_{k+1} &= f_k(x_k, w_k) \qquad k=0,\dots,N-1 \\ y_k &= g_k(x_k, w_k) + v_k \end{aligned}}$$

w_k: ~~NOISE~~ DISTURBANCES
v_k: MEAS. NOISE

ASS: f_k, g_k KNOWN
v_k, w_k UNCERTAIN, BUT STOCHASTIC PROPERTIES KNOWN
+ A-PRIORI: x_0 KNOWN STOCHASTIC PROPS.

$v_0, \ldots, v_{N-1}, \quad w_0, \ldots, w_{N-1}, \quad x_0$

INDEPENDENT

$p(v_n) \equiv$ PDF OF RANDOM VAR V_n TO TAKE VALUE v_n

$p_{v_n}(z)$

$\int p(v)\, dv = 1$

$p_{v_5}(\zeta)$

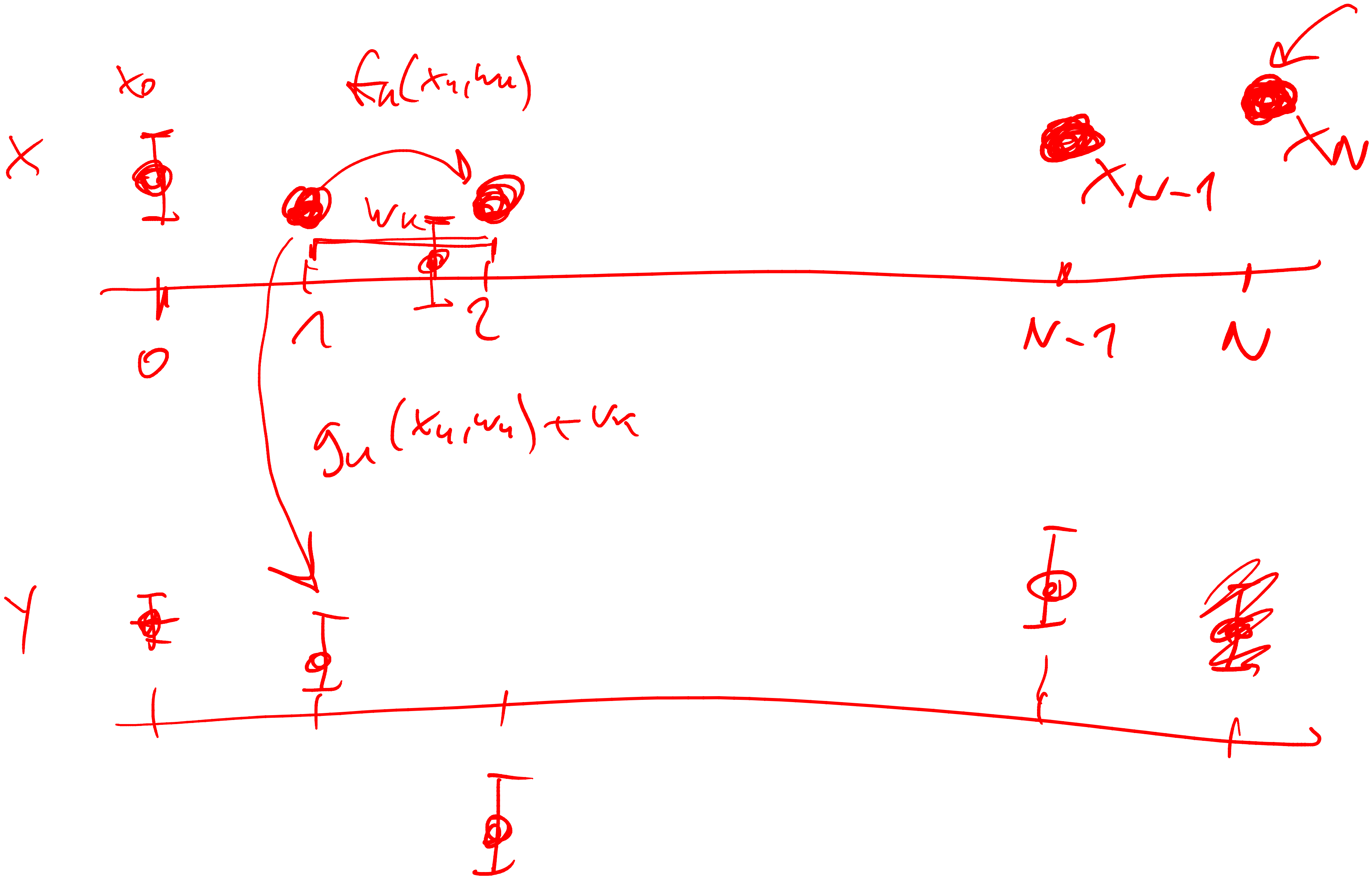
$p(w_5)$

$p(\zeta)$

$\beta_5(\zeta) =$

$$\boxed{\begin{aligned} p(v_n) &= \exp(-\phi_n(v_n)) \cdot \text{const} \\ & \quad \text{"PENALTY"} \\ p(w_n) &= \exp(-\beta_n(w_n)) \cdot \text{const} \\ p(x_0) &= \exp(-\alpha_0(x_0)) \cdot \text{const.} \end{aligned}}$$

$\phi_n = -\log p(v_n)$



X
Y
x_0
$f_k(x_k, w_k)$
w_k
x_{N-1}
x_N
0
1
2
N-1
N
$g_k(x_k, w_k) + v_k$

12.1.1 GENERALITY → PLEASE READ

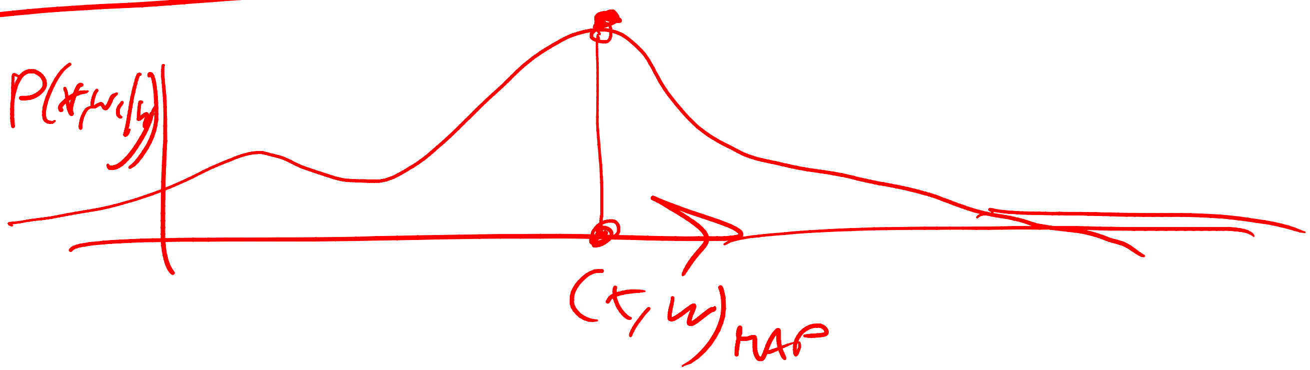
12.2 TRAJECTORY ESTIMATION PROBLEM

BAYESIAN FRAMEWORK: GIVEN $y = (y_0, \ldots, y_{N-1})$

FIND $(x_0, \ldots, x_N)$, $(w_0, \ldots, w_{N-1})$

THAT MAXIMIZE A POSTERIORI ~~PROB~~ LIKELIHOOD (MAP)

$$\boxed{\underset{x,w}{\operatorname{argmax}}\ \underline{P(x,w \mid y)} = \begin{pmatrix} x \\ w \end{pmatrix}_{MAP}}$$



~~MINI~~ ~~MICRO~~ NANO COURSE ON PROBABILITY

$P(A)$ PROB. OF EVENT A

$P(A \cap B)$ PROB TO GET BOTH A & B

$P(A,B)$

$P(A|B)$ PROB. OF EVENT A <u>GIVEN</u> B

$$\boxed{P(A|B) = \frac{P(A,B)}{P(B)}}$$

$$\boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$$

SAME FOR PDF

$$P(x,w|y) = \frac{P(y|x,w)\,P(x,w)}{P(y)}$$

B A

CONSTANT

A

$$P(x,w) = \begin{cases} 0 & \text{IF NOT } x_{n+1} = f_n(x_n, w_n)\ \forall_{n=0,\dots,N-1} \\ P(x_0)\cdot P(w_0)\cdot P(w_1)\dots P(w_{N-1}) & \text{ELSE} \end{cases}$$

$$-\log P(x,w) = \begin{cases} \infty & \text{IF NOT } x_{n+1} = f_n\dots \\ \alpha_0(x_0) + \sum_{n=0}^{N-1} \beta_n(w_n) \end{cases}$$

B $P(y|x,w) = \prod_{n=0}^{N-1} P(y_n|x_n,w_n) = \prod_{n=0}^{N-1} P(v_n)$

$$-\log P(y|x,w) = \sum_{n=0}^{N-1} \Phi_n(y_n - g_n(x_n,w_n))$$

$$\arg\max_{x,w} P(x,w\,|\,y) = \arg\max_{x,w} P(y\,|\,x,w)\cdot P(x,w)$$

$$= \arg\min_{x,w} -\log P(y\,|\,x,w) - \log P(x,w)$$

$$= \arg\min_{x,w} \alpha_0(x_0) + \sum_{n=0}^{N-1} \underbrace{\left[\beta_n(w_n) + \phi_n(y_n - g_n(x_n,w_n))\right]}_{\varphi_k(x_n,\,w_n)}$$

$$\text{s.t.} \quad x_{n+1} = f_n(x_n, w_n) \qquad k = 0, \dots, N-1$$

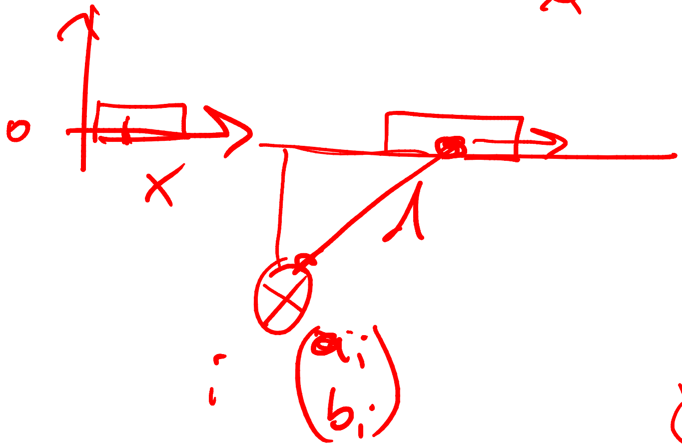
AN OCP!

$$\boxed{\begin{aligned} &\min_{\substack{x_0,\dots,x_N\\ w_0,\dots,w_{N-1}}} \quad \alpha_0(x_0) + \sum_{k=0}^{N-1} \varphi_k(x_k, w_k) \\ &\text{s.t.} \quad x_{k+1} = f_k(x_k, w_k), \quad k = 0,\dots,N-1 \end{aligned}}$$

PARTIALLY SEPARABLE LAGRANGIAN ETC.

NO TERMINAL COST, BUT "ARRIVAL COST" $\alpha_0(x_0)$

EX: 1D-ROBOT (TRAIN): MEASUREMENTS AT t_k: DISTANCES TO n LANDMARKS ⊗,



x_k POSITION AT t_k

s_k SPEED COMMAND BETWEEN $[t_k, t_{k+1}]$

⊗

$$\boxed{\begin{aligned} x_{k+1} &= x_k + (s_k + w_k)(t_{k+1} - t_k) \\ y_k^{[i]} &= \sqrt{(x_k - a_i)^2 + b_i^2} + v_k \end{aligned}}$$

$\phi = ?$

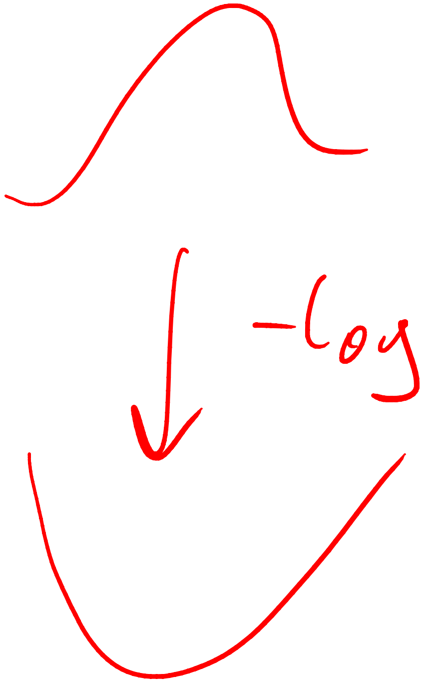
$\beta = ?$

$p(\overset{v_u}{\cancel{x_k}}) \sim \mathcal{N}(0, \sigma^2)$

$p(v_u) = \text{const} \cdot e^{-\frac{(v_u^2)}{2\sigma^2}}$

$\Phi_u(v_u) = -\log p(v_u)$

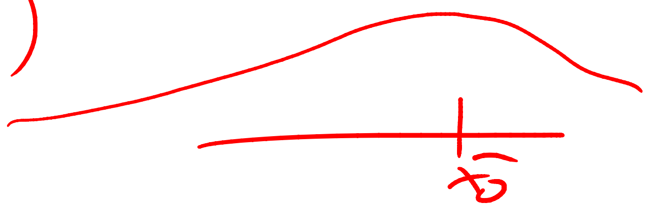
$= \frac{v_u^2}{2\sigma^2} + \text{const.}$



1 $w_u \sim \mathcal{N}(0, \sigma_w^2)$

2 $v_u \sim \mathcal{N}(0, \sigma_v^2)$

3 $x_0 \sim \mathcal{N}(\bar{x}_0, \sigma_{x_0}^2)$



1 $\beta_u(w_u) = \frac{w_w^2}{2\sigma_w^2}$

2 $\Phi_u(v_u) = \frac{v_u^2}{2\sigma_v^2}$

3 $\alpha_0(x_0) = \frac{(x_0 - \bar{x}_0)^2}{2\sigma_{x_0}^2}$

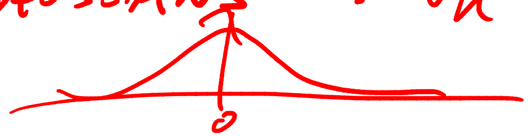
$$\min_{x,v} \quad \frac{(x_0-\bar{x}_0)^2}{2\sigma_{x_0}^2} + \sum_{k=0}^{N-1}\left[\frac{w_k^2}{2\sigma_w^2} + \sum_{i=1}^{M}\frac{\left(y_k^{[i]} - \sqrt{(x_k-a_i)^2+b_i^2}\right)^2}{2\sigma_v^2}\right]$$

$$\text{s.t.} \quad x_{k+1} = x_k + (s_k + w_k)(t_{k+1} - t_k) \qquad k=0,\dots,N-1$$

EXAMPLES FOR STAGE COSTS

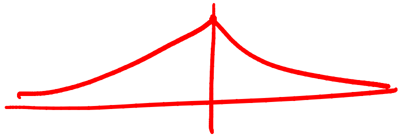
CH12.2.1

a) GAUSSIANS : $v_k \sim \mathcal{N}(0, \Sigma_v) \longrightarrow \Phi_k(v_k) = v_k^T \Sigma_v^{-1} v_k$

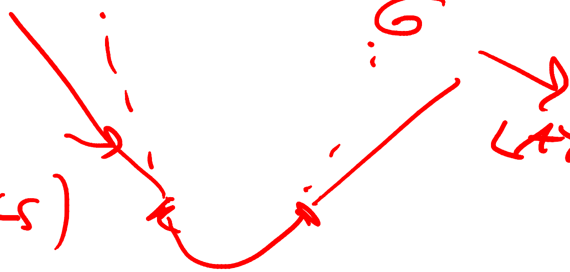


WITH $\Sigma_v \succ 0$

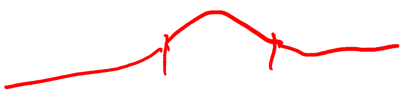
b) LAPLACE-DISTRIB.



$p(v_k) = e^{-\frac{|v_k|}{\sigma}} \longrightarrow \Phi_k(v_k) = \frac{|v_k|}{\sigma} \longleftarrow$



c) HUBER-PENALTY (GAUSSIAN WITH FAT TAILS)



LATER