OPTIMAL CONTROL & ESTIMATION

- DYNAMIC SYSTEMS
- OPTIMIZATION

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50% OF 9 FROM 11 SHEETS

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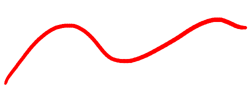
CH 1 : INTRODUCTION

- SYSTEM MODEL
- INPUTS / CONTROLS u
- OBJECTIVES
- CONSTRAINTS
  - EQUALITY
  - INEQUALITY

1.1 DYNAMIC SYSTEMS



$\dot{x} = f(x, u)$ ODE , CONTINUOUS TIME

$x_{k+1} = f_{dis}(x_k, u_k)$ DIFFERENCE EQUATION, DISCRETE TIME

- CONTINUOUS VS. DISCRETE STATE SPACES

  $x \in \mathbb{R}^n$

  $x \in \mathbb{Z}^n$

  (NOT TREATED HERE, ONLY IN DYN. PROG. CHAPTER)

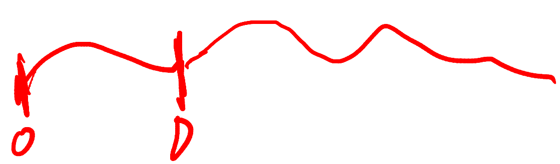
- FINITE VS INFINITE DIMENSIONAL STATE SPACES

  $x \in \mathbb{R}^n$

  X IS A FUNCTION (IN SPACE, ...)
  INFINITELY MANY NUMBERS, AT EVERY TIME POINT.

  EX: • TEMP. DISTRIBUTION
  • TIME DELAY IN CONTINUOUS TIME

  $$\dot{x} = f(x(t), x(t-D))$$



- CONT. VS. DISCRETE CONTROLS

(DYN. PROG.)

- TIME-VARIANT VS. TIME-INVARIANT SYSTEMS

$$\dot{x}(t) = f(x(t), u(t), \cancel{t}^{\tilde{J}}) \qquad \boxed{\dot{x}(t) = f(x(t), u(t))}$$

"CLOCK STATE" $\dot{\tilde{J}}(t) = 1 \qquad \tilde{J}(0) = 0$

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ \tilde{J}(t) \end{bmatrix}, \quad \tilde{f}(\tilde{x}(t), u(t))$$

- LINEAR VS. NONLINEAR SYSTEMS

$x = A \cdot x + Bu \qquad \dot{x} = f(x, u)$

LINEAR TIME INVARIANT (LTI)

$$\boxed{\dot{x}(t) = A \cdot x(t) + B \cdot u(t)}$$

A, B FIXED

$A \in \mathbb{R}^{n_x \times n_x}$

$B \in \mathbb{R}^{n_x \times n_u}$

LIN. TIME VARIANT (LTV)

$$\dot{x}(t) = A(t) \cdot x(t) + B(t) \cdot u(t)$$

CONTROLLED VS. UNCONTROLLED SYSTEMS

$$\dot{x} = f(x)$$

"AUTONOMOUS" $= \begin{cases} \text{UNCONTROLLED} \\ \text{TIME-INVARIANT} \end{cases}$

OPEN-LOOP VS. CLOSED-LOOP CONTROLS

$u(t)$

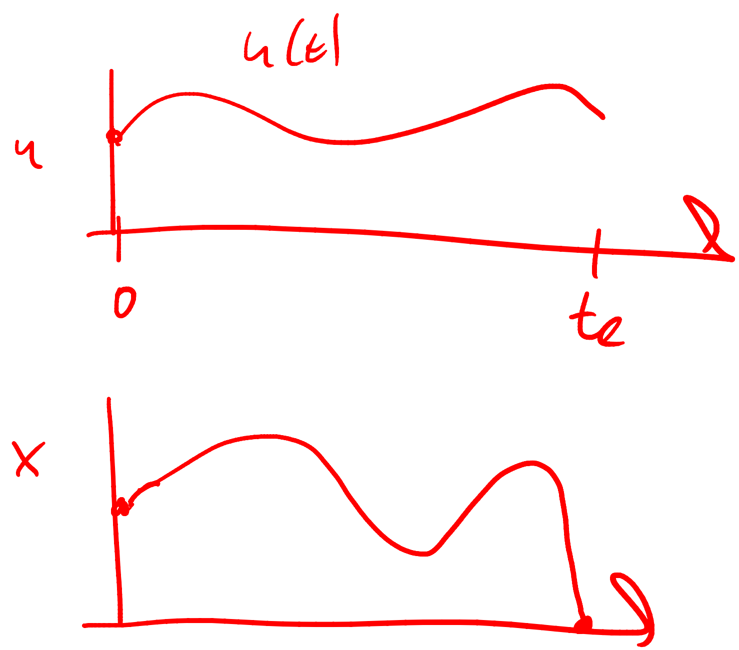
FEEDBACK CONTROL

$u(x(t))$

FIRST PART

DYN. PROG.

NMPC



FOCUS:
- CONT. STATES & CONTROLS
- DISCR. TIME / CONT.
-

• STABLE VS UNSTABLE SYSTEMS

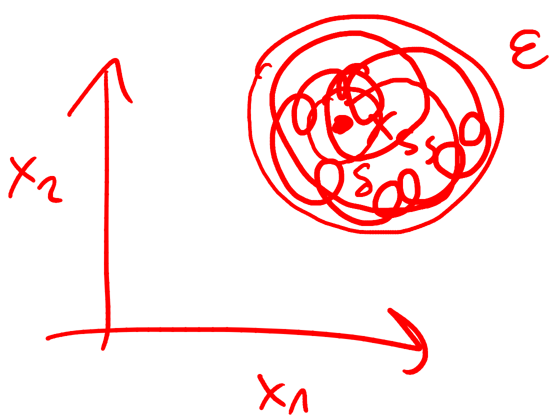
EASIEST FOR AUTONOMOUS SYSTEMS :

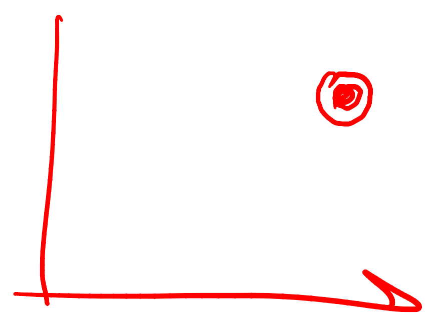
$$\boxed{\boxed{\dot{x} = f(x)} \quad , \quad x_{ss} \quad \text{WITH} \quad f(x_{ss}) = 0} \rightarrow x(0) \searrow x(t)$$

x_{ss} IS STABLE IF FOR ANY $\varepsilon > 0$ $\exists\, \delta > 0$

$\forall\, x(0) : \|x(0) - x_{ss}\| \le \delta \; : \; \|x(t) - x_{ss}\| \le \varepsilon \quad \forall\, t \ge 0$

(LYAPUNOV-STABILITY)





• DETERMINISTIC VS. STOCHASTIC SYSTEMS

$\dot{x} = f(x)$

$x_{k+1} = f_{dis}(x_k)$.

(ITÔ-CALCULUS)

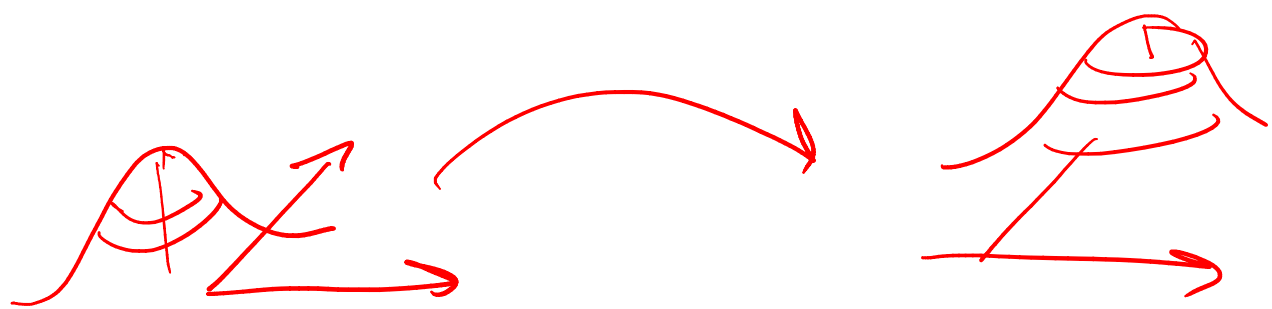
$x_{k+1} = f_{dis}(x_k, w_k)$

w_k IS STOCHASTIC VARIABLE

MARKOV-CHAINS (DISCRETE STATE SPACES)

$p(x_{k+1} | x_k)$

MHE



1.2 CONTINUOUS TIME SYSTEMS

ORDINARY DIFFERENTIAL EQ (ODE)

$\dot{x} = f(x, u, t)$ WITH OPEN-LOOP CONTROLS $u(t)$

IVP $\left\{ \begin{array}{l} \boxed{\dot{x} = f(x, t)} \quad t \in [t_{init}, t_{fin}] \\ \boxed{x(t_{init}) = x_{init}} \end{array} \right.$

TIME VARIANT, UNCONTROLLED

$\dot{x} = f(x, u)$ $\quad \tilde{f} = f(x, u(t))$

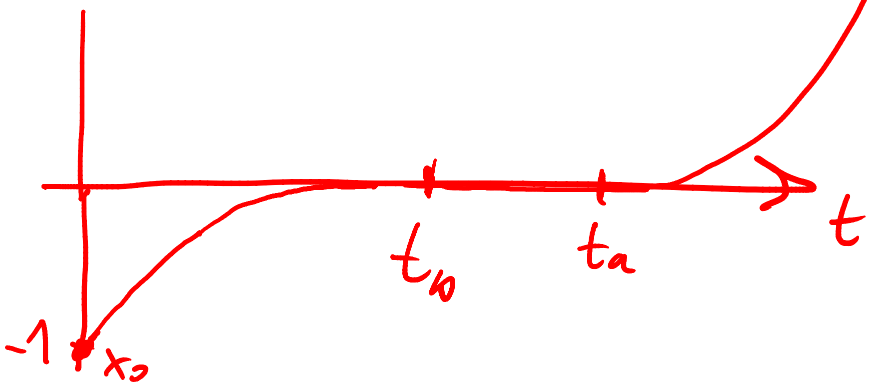
$\dot{x} = \tilde{f}(x, t)$

INITIAL VALUE PROBLEM (IVP)

$x(t)$ IS UNIQUELY DEFINED ?

NOT ALWAYS: $\qquad x \in \mathbb{R}$

$$\boxed{\dot{x} = \sqrt{|x|}} \qquad x_0 = -1$$



$$x(t) = \frac{(t-t_a)^2}{c}$$

$$\dot{x}(t) = \frac{2(t-t_a)}{c}$$

$\parallel$?

$$\sqrt{|x(t)|} = \frac{t-t_a}{\sqrt{c}}$$

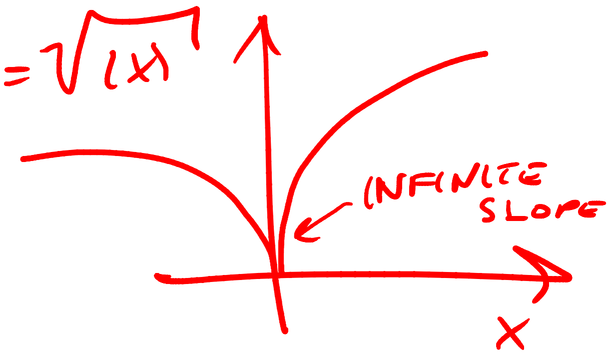
$$\frac{2}{c} = \frac{1}{\sqrt{c}} \qquad \boxed{c = 4}$$

$$x(t) = \frac{(t-t_a)^2}{4} \quad \text{FOR ANY } t_a$$

$$x(t) = 0$$

$$x(t) = -\frac{(t-t_0)^2}{4}$$

$$f(x) = \sqrt{|x|}$$



THM 1 (PICARD LINDELÖF)

IF: $f: \mathbb{R}^{n_x} \times [t_{init}, t_{fin}] \to \mathbb{R}^{n_x}$

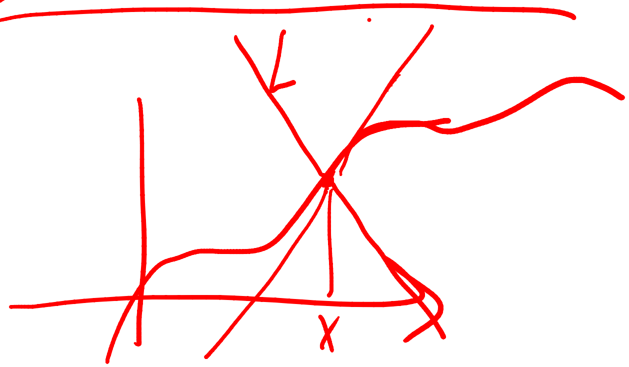
$\dot{x} = f(x,t)$, $x(t_{init}) = x_{init}$

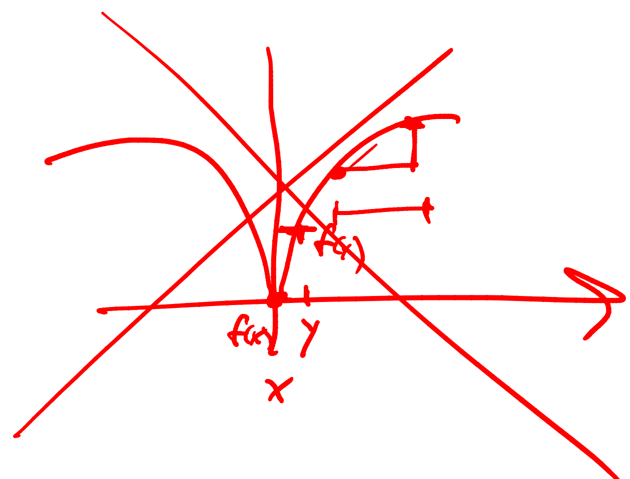
$\exists L < \infty$ $\boxed{\|f(x,t) - f(y,t)\| \le L \cdot \|x - y\|}$ $\forall t \in [t_{init}, t_{fin}]$, $y, x \in \mathbb{R}^{n_x}$,

THEN: $x(t)$ $t \in [t_{init}, t_{fin}]$ EXISTS AND IS UNIQUE

LIPSCHITZ - CONDITION

SLOPE OF F IS BOUNDED





$f(x) = x^2$

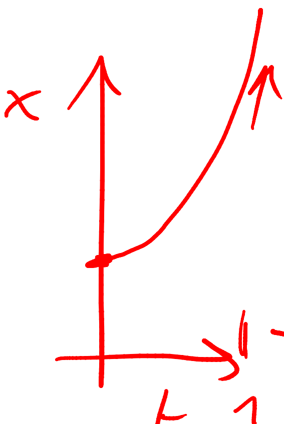
$$\boxed{\dot{x} = x^2}$$

$x(0) = 1$

$$\frac{dx}{dt} = x^2$$

$$\frac{dx}{x^2} = dt$$

$$\int_0^t x^{-2}\,dx = \left[-x^{-1}\right]_0^t$$



$$-x(t)^{-1} + x(0)^{-1} = t$$

$$x(t) = \left(x(0)^{-1} - t\right)^{-1}$$

$$= (1-t)^{-1} = \frac{1}{1-t}$$

EX: LTI SYSTEMS, WITH OPEN-LOOP CONTROLS

$$\cancel{\dot{x} = A\cdot x + Bu} \qquad \cancel{f(x) = A\cdot x}$$

$$\dot{x} = A\cdot x + Bu(t) \qquad f(x,t) = A\cdot x + \underbrace{(B\cdot u(t))}_{g(t)}$$

$$f(x,t) - f(y,t)$$

$$= Ax + g(t) - Ay - g(t)$$

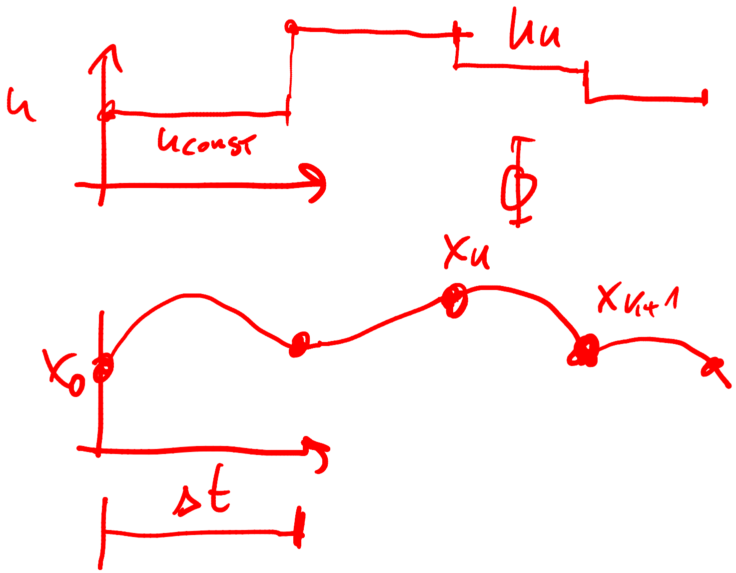
$$= A(x-y)$$

$$\|f(x,t) - f(y,t)\| = \|A(x-y)\| \leq \|A\|\cdot\|x-y\|$$

$$L = \|A\| < \infty$$

LTI SYSTEMS HAVE UNIQUE SOLUTIONS ON ARBITRARY HORIZONS

ZERO ORDER HOLD AND SOLUTION MAP



PIECEWISE CONSTANT CONTROLS

$$\dot{x} = f(x, u_{const})$$

$x(t+\Delta t)$ FROM $x(t)$, u_{const}

FROM CONT. TIME CONTROLLED SYSTEM $\dot{x} = f(x, u)$

$$\boxed{x_{k+1} = \Phi(x_k, u_k)}$$